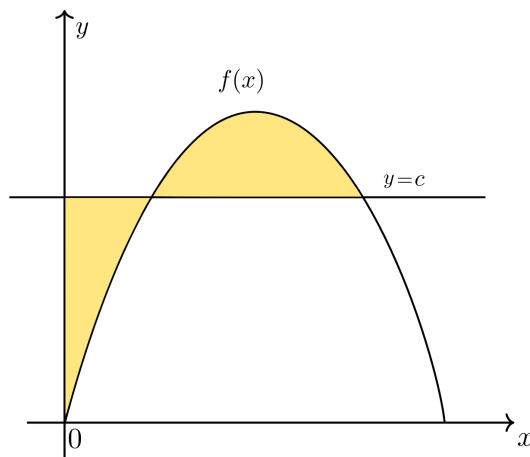


Equal Areas: Putnam 1993 A1

Name & Period: _____ Date: _____



Problem 226.¹ The horizontal line $y = c$ intersects the curve $f(x) = 2x - 3x^3$ in the first quadrant as in the figure above. Find the value of c so that the areas of the two shaded regions are equal.

Directions. What is to follow is a guide of scaffolded components of the problem. Don't think of it as strict. At the end of the day, the goal is to solve the problem with a comprehensive argument and write a program that actually works.

Part I: Describe the Problem in Detail

1. Describe how the left-most yellow region is bounded by the curves.

2. Describe how the right-most yellow region is bounded by the curves.

¹This problem was first published in *The William Lowell Putnam Mathematical Competition 1985–2000: Problems, Solutions, and Commentary*, ©2008 MAA (Mathematical Association of America).

Solutions.

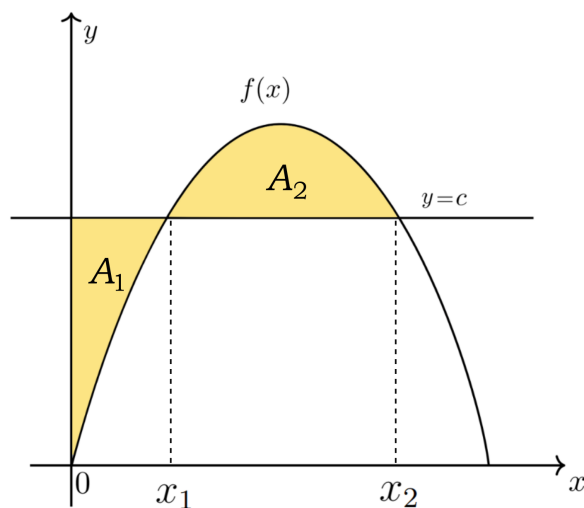
The Desmos program (the solution to the challenge problem) can be given at the beginning of the project as a conceptual guide.

Part I: Describe the Problem in Detail

1. **Solution.** This region is bounded above by the line $y = c$, bounded on the left by $x = 0$ (the y -axis), and bounded on the right by $f(x) = 2x - 3x^3$.
2. **Solution.** This region is bounded below by the line $y = c$ and bounded above by the curve $f(x) = 2x - 3x^3$.
3. **Solution.** We can use integral calculus if we can describe the curve in terms of a function. Though, there are caveats to this principle. Take for example Archimedes finding the volume of a sphere: he did not use an integral explicitly, but he did use some form of calculus (a.k.a. the *method of exhaustion*).

Part II: Area

4. **Solution.**



5. **Solution.** It is the area under the top curve minus the area under the bottom curve.

$$A_1 = \int_0^{x_1} (c - f(x)) dx$$

6. **Solution.** It is the area under the top curve minus the area under the bottom curve.

$$A_2 = \int_{x_1}^{x_2} (f(x) - c) dx$$