Maximum Area of Rectangle Inscribed Inside Triangle

Name & Period:______________________________ Date:__________________

Problem 90. (from the book *Learning Isn't Linear*) See the diagram below. What is the rectangle of largest area that can be inscribed in the triangle such that one side lies along the x-axis and the other side lies along the y-axis? Also, write a Desmos program that finds the area of the largest rectangle.

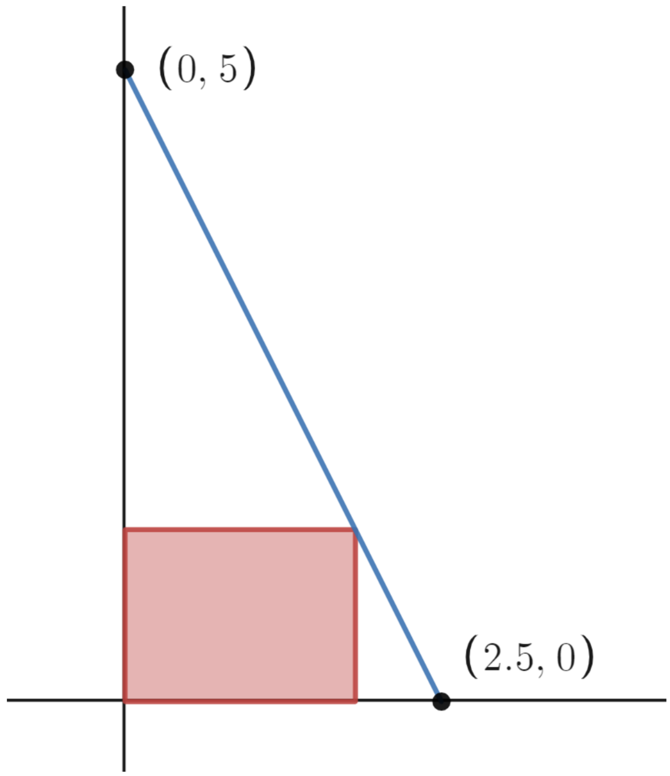

Figure 1: Rectangle inscribed in triangle.

Part I: Play & Experiment

Construct three different rectangles inscribed in the triangle and find their areas. To do this, first find the equation of the line that contains the points $(0, 5)$ and $(2.5, 0)$.

1. Find the equation of the line.

Work:

Solutions.

Part I.

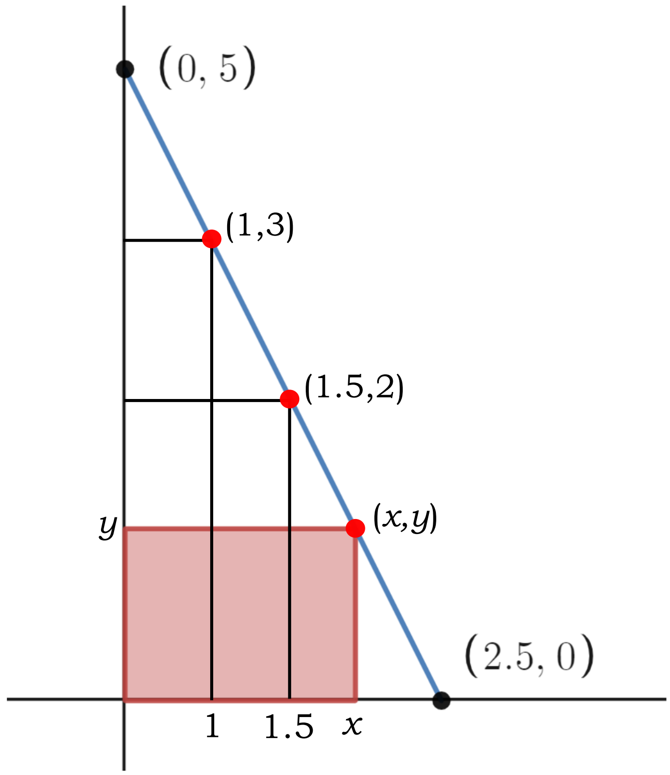

Figure 2: Rectangle inscribed in triangle.

1. The equation of any straight line can be put in the form $y = mx + b$. Find the slope first. By definition, the slope

$$m = \frac{\text{rise}}{\text{run}} = -\frac{\text{length of vertical leg of triangle}}{\text{length of bottom leg of triangle}} = -\frac{5}{2.5} = -2.$$

Then, $b = 5$ since the y-int is $(0, 5)$. Thus, the equation of the line is

$$y = -2x + 5.$$

Note. Of course, one can alternatively plug in the points to obtain slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 0}{0 - 2.5} = -2.$$

2. We have many choices for rectangles. The area A of any rectangle is given by $A = LW$ (Length$\times$Width). Using the equation of the line, we observe the x-coordinate is the width and the y-coordinate is the length/height. Therefore, if we pick $x = 2$, then $y = -2(2) + 5 = 1$, and the area of Rectangle #1 will be $A = 2 \cdot 1 = 2$.